Research Article

Systematic Versus Idiosyncratic Risk in Stress Testing Systematic

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Abstract: The Comprehensive Capital Analysis and Review is an annual exercise by the Federal Reserve to assess whether the largest bank holding companies have sufficient capital to continue operations under adverse scenarios for the systematic component. In this paper, we extend Vacisek's framework by modeling an equally probable stress scenario (a 99.9th percentile event) that can result from either the systematic or the idiosyncratic component. From this composite function, we then select a stress scenario that satisfies two conditions. First, it corresponds to a 99.9th percentile event and, second, it maximizes the conditional probability of default at the portfolio level. The maximization choice varies with asset correlation and portfolio diversification assumptions.

Keywords: Credit, Probability of Default, Stress Testing, Systematic, idiosyncratic.

1. INTRODUCTION

The main determinant of capital requirements is credit risk, which generally arises from a counterparty failing to meet a repayment commitment on its outstanding debt. Such credit events, however, tend to correlate with adverse macroeconomic (systematic) events. The measurement of credit risk under adverse scenarios is thus a main objective in enterprise risk management (Sarmiento, 2017). For example, regulators and senior management of major US banks rely on stress testing to guarantee that the bank holding company has sufficient capital to continue operations throughout times of economic and financial stress.

The determination of capital requirements for credit risk hinges on loss forecasts. For example, probability of default (PD) and loss given default (LGD) models provide the framework for projection of losses under different adverse scenarios that include rare events. Such projections can then be used to determine capital levels that ensure solvency of the bank under different levels of stress.

The use of agency ratings (e.g., Standard & Poor's) in the determination of capital requirements is a common practice for projecting losses of portfolio classes that have little or no default history. In these instances, the credit rating of each counterparty can be mapped to an average probability of default per rating. Such data is available for over 30-years of data and it is reported by rating agencies (e.g., see S&P Global Ratings, 2016). Therefore, if a bank has a portfolio with few default events, it can still assign a probability of default to each rated counterparty.

An average probability of default, however, provides little help in the determination of capital requirements. In a seminal contribution, Vacisek (2002) solves the puzzle by mapping the unconditional (average) probability to a conditional probability of default. For example, the conditional probability of default can be defined to subsume a stress event, a 99.9th percentile event for the systematic (macroeconomic) component that drives default. A main assumption in the mapping is that the systematic component is sufficient to capture capital requirements. Idiosyncratic risk can, thus, be ignored. This assumption dominates stress testing. For example, the Comprehensive Capital Analysis and Review is an annual exercise by the Federal Reserve to assess whether the largest bank holding companies have sufficient capital to continue operations under adverse scenarios for the systematic component.

In this paper, we directly test the assumption that the systematic component dominates the idiosyncratic component.[1] To do so, we extend Vacisek framework by modeling a stress scenario (a 99.9th percentile event) that can result from either the systematic or the idiosyncratic component. From this composite function, we then select a stress scenario that satisfies two conditions. First, it corresponds to a 99.9th percentile event and, second, it maximizes the conditional probability of default at the portfolio level. The maxi-

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[1] In contrast, Gordy (2004) introduces idiosyncratic risk as the additional risk associated with a second order Taylor expansion around the idiosyncratic component and evaluated as an add-on to the 99.9th stress percentile of the systematic component. The approach, therefore, incorporates the additional fraction of risk from the idiosyncratic component to Vacisek formula. The Basel Committee on Banking Supervision (2014) defines applications in which such add-on is needed for regulatory capital.

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mization choice varies with asset correlation and portfolio diversification assumptions.

2. CONDITIONAL PROBABILITY OF DEFAULT BASED ON SYSTEMATIC RISK

The use of agency ratings in the determination of capital requirements is a common practice for projecting losses of portfolio classes that have little or no default history. In these instances, the credit rating of each obligator can be mapped to an unconditional (average) probability of default per rating. To determine capital requirements needed to cover unexpected losses for obligator i, Vacisek (2002) maps the unconditional probability of default to a conditional probability.

In particular, Vacisek's mapping assumes that the asset value of a given obligator i has a stochastic component, X_{it}, that follows the stochastic process:

$$X_{it} = S_t\sqrt{p} + Z_{it}\sqrt{1-p} \tag{1}$$

and

S_t~N(0,1) and Z_{it} ~N(0,1)

where the systematic component is S_t; the idiosyncratic component is Z_{it}; and the asset correlation across obligators is p. Generally, S_t relates to an economic index relevant to the obligator economic sector, and the asset correlation, p, determines the relative importance of the systematic component.[2] Empirical estimates of asset correlation range from .10 to .25 (e.g., see Zhan, Zhu, and Lee, 2008), which underscore the larger role of the idiosyncratic component in explaining default at the obligator level.[3]

Consistent with Merton (1977), the default event occurs if the random component of the stochastic component X_{it} falls below a threshold C. Therefore, the unconditional probability of default (PD) is:

$$P(X_{it} < C) = [\varphi^{-1}(C)] = PD \tag{2}$$

where φ is the distribution function of S_t.

As a Corollary, the conditional probability of default (CPD) for a given realization of the systematic component S_t is:

$$CPD = P(X_{it} < C|S_t) = \varphi[(\varphi^{-1}(\text{PD}) - S_t\sqrt{p})/\sqrt{1-p}] \tag{3}$$

To express the CPD in terms of the probability of the systematic component, Equation 3 is first solved in terms of S_t as follows:

$$\varphi(-S_t) = \varphi[\{\varphi^{-1}(CPD)\sqrt{1-p} - \varphi^{-1}(\text{PD})]/\sqrt{p}\}] \tag{4}$$

2 Reinhart and Rogoff (2005) underscore the role of systematic factor in repeated sovereign default events.

3 Hilscher and Wilson (2016) estimates that the idiosyncratic component explains up to 80 percent of the firm level default probabilities.

where $\varphi(S_t)$=1- φ (S_t). Secondly, solving Equation 4 in terms of the CPD yields:

$$CPD = \varphi[\{\sqrt{p}\,\varphi^{-1}(1 - \varphi(S_t)) + \varphi^{-1}(\text{PD})\}/\sqrt{1-p}] \tag{5}$$

The CPD representation in Equation 5 is solely a function of the probability of the associated systematic component, the PD, and the asset correlation. Therefore, under the assumption that all obligators share the same PD, the equation represents both the CPD at the obligator level and the portfolio level.

As a result, a main assumption of Vacisek's mapping in Equation 5 is that the systematic component is sufficient to capture capital requirements. Idiosyncratic risk can, thus, be ignored.

The main objective of the CPD is to capture a stress scenario, and the probability of the systematic component in Equation 5 comprises the basis for defining the CPD under stress. Specifically, if we set an adverse event for S_t to correspond to a 0.1 percent event (a 99.9th percentile event), then the CPD corresponds to the probability of default that underpins risk weights under the Basel formula for regulatory capital (see Bank of International Settlements, 2005).

Next, we extend Vacisek framework by modeling a stress scenario (0.1 percent event) associated with the idiosyncratic component. The stressed PD under the systematic component is then compared to an equally stressed probability event but under the idiosyncratic component.

3. CONDITIONAL PROBABILITY OF DEFAULT BASED ON IDIOSYNCRATIC RISK

From Equations 1 and 2, the CPD for a given realization of the idiosyncratic component, Z_t, is:

$$CPD_i^* = P(X_{it} < C|Z_{it}) = \varphi[(\varphi^{-1}(\text{PD}) - Z_{it}\sqrt{1-p})/\sqrt{p}] \tag{6}$$

Analogously to Equation 3, Equation 6 is solved in terms of the probability of Z_t as follows:

$$\varphi(-Z_{it}) = \varphi[\{\varphi^{-1}(CPD_i^*)\sqrt{p} - \varphi^{-1}(\text{PD})\}/\sqrt{1-p}] \tag{7}$$

Further solving Equation 7 in terms of CPD^*_i yields the CPD for obligator i defined in terms of the probabilistic realization of the idiosyncratic component:

$$CPD_i^* = \varphi[\{\sqrt{1-p}\,\varphi^{-1}(1 - \varphi(Z_{it})) + \varphi^{-1}(\text{PD})\}/\sqrt{p}]. \tag{8}$$

Equation 8, therefore, subsumes the case of an adverse event for the idiosyncratic component, Z_{it}. Consistent with the definition of stress used for the systematic component in Equation 5, we assume that the adverse event in Equation 8 corresponds to a .1 percent event.

Equation 8 is particularly important in the evaluation of credit risk. For example, Hilscher and Wilson (2016) estimate that the idiosyncratic component comprises the largest component of credit risk at the obligator level. This finding is consistent with empirical asset correlation estimates that range from .1 to .25 in the context of Equation 1. Unlike the CPD defined in terms of the systematic component, the CPD defined in terms of the idiosyncratic component changes at the portfolio level relative to the obligator level.

To derive the CPD at the portfolio level associated with Equation 8, we assume without much loss of generality that the portfolio consists of identical and equally proportional borrowers. Furthermore, the portfolio is comprised of n obligators and each obligator follows the stochastic process in Equation 1. As in Equation 5, the CPD is associated with an event that has a .1 percent probability. Unlike Equation 5, the adverse occurrence relates to the idiosyncratic component.

Under these simplifying assumptions, the CPD at the portfolio level associated with a 99.9th percentile event for the idiosyncratic component is:

$$CPD^* = \frac{1}{n} \underset{Z_{1t},..Z_{nt}}{Max} \sum_{i=1}^{n} \varphi[[\{\sqrt{1-p}\varphi^{-1}\left(1-\varphi(Z_{it})\right) + \varphi^{-1}(\text{PD})\}/\sqrt{p}]]$$

$$\left|\left\{\begin{pmatrix} 1-\varphi(Z_{1t}) \times \ldots. \\ 1-\varphi(Z_{nt}) \end{pmatrix} = .001\right\}\right] \quad (9)$$

where the maximization in Equation 9 depends on the number of borrowers, the asset correlation, and the PD. By definition, Equation 9 captures an equally probable event than Equation 5. Different from the systematic component, however, the level of stress under the idiosyncratic component depends on the number of obligators.

Solving Equation 9 requires the use of numerical procedures. The application of such procedure finds that the maximum risk occurs when the worst event concentrates on one obligator. Alternatively, the equation is minimized when the idiosyncratic realization of the risk is evenly distributed across obligators. For example, for a portfolio with two obligators, the equation is maximized when one obligator experiences a 99.8th event while the second obligator experiences an average idiosyncratic event. To further illustrate how portfolio risk varies with the distribution of idiosyncratic risk, consider five obligators. In this instance, Equation 9 is maximized when one obligator experiences a 96.8th event while the other obligators experience average idiosyncratic events. The distribution of probabilities that encompasses the 99.9th percentile event becomes irrelevant for exposures with more than 10 borrowers.

Therefore, we can numerically solve Equation 9 and obtain the following result:

RESULT 1: If credit exposures encompass less than ten (equi-proportional) obligators, it then follows that the solution of Equation 9 is:

$$CPD^* = \frac{1}{n}\varphi[\{\sqrt{1-p}\varphi^{-1}(1-s_n) + \varphi^{-1}(\text{PD})\}/\sqrt{p}] + \frac{1}{n}\sum_{i=1}^{n-1}\varphi[\{\sqrt{1-p}\varphi^{-1}(1-.5) + \varphi^{-1}(\text{PD})\}/\sqrt{p}] \quad (10)$$

where $s_n \times .5^n = .001$.

This result also applies if a single obligator encompasses more than 10 percent of the overall exposure at the institution.

However, if credit exposures encompass more than 10 (equi-proportional) obligators, then the risk associated with the idiosyncratic component becomes quite small, and the solution of Equation 9 can be represented as:

$$CPD^* = \frac{1}{n}\sum_{i=1}^{n}\varphi[\{\sqrt{1-p}\varphi^{-1}(1-.001^{\frac{1}{n}}) + \varphi^{-1}(\text{PD})\}/\sqrt{p}] \quad (11)$$

4. A TEST OF DOMINANCE OF SYSTEMATIC VERSUS IDIOSYNCRATIC RISK

The previous section derived an alternative measure of the conditional probability of default. Next, we illustrate conditions under which the idiosyncratic component dominates over the systematic component for a 0.1 percent adverse event. That is,

Dominant CPD = *Max*(*CPD*,*CPD**)

Table **1** shows the CPD for different asset correlation assumptions and varying number of obligators while assuming that the unconditional PD is 1 percent. Table **1** is based on ranges of empirically derived asset correlation per sector (see Zhan j, Zhu, F, and Lee, J, 2008; the Bank of International Settlements, 2005). For example, the asset correlation for sovereigns is often assumed to be .2 while the asset correlation for industrial sectors range from .1 to .25. The results in Table 1 thus depend on the sector.

Testing results in Table **1** indicate that the CPD defined in terms of the systematic component dominates the idiosyncratic component as long as the largest obligator does not exceed 25 percent of the portfolio. The importance of the idiosyncratic component is larger as the asset correlation tends to zero.

Therefore, in most instances, the Vacisek model is accurate in capturing the effect of a 99.9th percentile event on the probability of default. As shown in Table **1**, however, the idiosyncratic component dominate the systematic component for portfolios with a small number of obligators, an exception to Vacisek model.

5. CONCLUSION

In this paper, we extended Vacisek's framework by modeling an equally probable stress scenario that can result from either the systematic or the idiosyncratic component. From this composite function, we then selected a stress scenario that satisfies two conditions. First, it corresponded to a 99.9th percentile event and, second, it maximized the conditional probability of default at the portfolio level. The maximization choice varies with asset correlation and portfolio diversification assumptions, and we identified that in highly concentrated portfolios the idiosyncratic component dominates over the systematic component in stress testing, an exception to Vacisek model.

Table 1. Probability of Default Conditional to a 99.9th percentile Event

Stress Event	Number of Borrowers	Share Largest Obligator	Asset Correlation			
Idiosyncratic Component 99.9th Percentile Event			0.1	0.15	0.2	0.4
	1	100%	97%	91%	84%	54%
	2	50%	45%	41%	35%	22%
	3	33%	25%	21%	18%	12%
	4	25%	12%	11%	10%	7%
	5	20%	5%	5%	5%	4%
	6	17%	2%	2%	2%	2%
	8	12%	1%	1%	1%	1%
	Number of Borrowers	**Share Largest Obligator**	**Asset Correlation**			
			0.1	0.15	0.2	0.4
Systematic Component 99.9th Percentile	1	100%	7%	11%	14%	31%
	2	50%	7%	11%	14%	31%
	3	33%	7%	11%	14%	31%
	4	25%	7%	11%	14%	31%
	5	20%	7%	11%	14%	31%
	6	17%	7%	11%	14%	31%
	8	13%	7%	11%	14%	31%

We find that the CPD defined in terms of the systematic component dominates the idiosyncratic component as long as the largest obligator does not exceed 25 percent of the portfolio. The importance of the idiosyncratic component is larger as the asset correlation tends to zero. Therefore, the idiosyncratic component may dominate the systematic component in the definition of a stress event only for certain small regional lenders as well as for some multilateral institutions. Lastly, the results of the paper can be easily extended to actual portfolios that embed different PDs across obligators.

CONFLICT OF INTEREST STATEMENT

The authors declare that they have no conflict of interest.

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